UUCMS No. | | | | | | | | | | | |

B.M.S COLLEGE FOR WOMEN, AUTONOMOUS

BENGALURU -560004

END SEMESTER EXAMINATION – APRIL/ MAY 2023

M.Sc. Mathematics – III Semester

NUMERICAL ANALYSIS- II

Course Code MM305T **QP Code: 13005**
Duration: 3 Hours **Max. Marks: 70**

Instructions: 1) All questions carry equal marks.
2) Answer any five full questions.

1. (a) Establish Picard's iteration method for $y' = f(x, y),\ y(x_0) = y_0$. Find the first four successive non-zero approximation of $y' = x + y, y(0) = 1$ and obtain $y(0.5)$.
(b) Obtain an approximation for $y' = x - y^2,\ y(0) = 1$ at $x = 0.1$ by finding five non-zero terms in the Taylor series. **(7+7)**

2. (a) Show that the Range-Kutta methods are relatively stable.
(b) Solve the following system of differential equations using second order Runge Kutta method: $\frac{dy}{dx} = z + 1, \frac{dz}{dx} = y - x; y(0) = 1, z(0) = 1$ for $x = 0.2$, take $h = 0.1$. **(7+7)**

3. (a) Derive Adam Bashforth third order method to solve the differential equation $y' = f(x, y)$,
subjected to the condition $y(x_0) = y_0$.
(b) Solve the initial value problem $y' = -2xy^2$, y(0)=1 with $h = 0.2$ on the interval [0,0.4], using the predictor -corrector method. **(7+7)**

4. (a) Solve the boundary value problem $y'' = xy,\ y(0) + y'(0) = 1, y(1) = 1$ with $h = \frac{1}{3}$ by using finite difference method.
(b) Solve $y'' + y' + 1 = 0, 0 \le x \le 1, y(0) = 0, y(1) = 0$, with $h = 0.5$ using Cubic spline methods. **(7+7)**

5. (a)Solve the Laplace equation $u_{xx} + u_{yy} = 0, 0 \le x, y \le 1$ subject to the condition $u(x, 0) = x, u(x, 1) = 0, u(0, y) = 0, u(1, y) = 0$. Choose $\triangle x = \triangle y = \frac{1}{3}$, using five point formula.

(b) Solve the Poisson equation $u_{xx} + u_{yy} = x^2 + y^2$ with $0 \le x, y \le 1$ and $u = 0$ on the boundary of the unit square, choose $\triangle x = \triangle y = \frac{1}{3}$. **(7+7)**

6. Derive the ADI method and show that the method is unconditionally stable. **(14)**

7. (a) Solve $\frac{\partial^2 u}{\partial t^2} = 4\ \frac{\partial^2 u}{\partial x^2}$ explicitly with the initial and boundary conditions $u(x, 0) = x^2(1 - x^2), u_t(x, 0) = 0, u(0, t) = u(1, t) = 0$ with $\triangle x = \frac{1}{4}, \triangle t = \frac{1}{64}$. Obtain the solution at first time level.
(b) Explain Crank Nicolson method and prove that stability on this method is unconditionally stable. **(7+7)**

8. Find the solution of $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}, 0 \le x, y \le 1$ with conditions $u(x, y, 0) = sin\pi x\ sin\pi y$; $\frac{\partial u}{\partial t} = 0$ and $u = 0$ on the boundary. Take $\triangle x = \triangle y = \frac{1}{2},\ \triangle t = \frac{1}{4}$. Perform one time integration using first Lee's alternating direction implicit method. **(14)**

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